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Solution existence of the optimization problem of truss structures with frequency constraints

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Abstract

A basic theory is presented for determining the solution existence of frequency optimization problems for truss structures. This theory says that the natural frequencies remain unchanged when a truss is modified uniformly and that the natural frequency constraint is usually the key constraint in determining the solution existence of a truss dynamic optimization problem. Based on this theory, a practical method is presented, in which only the first order derivatives of certain eigenvalues with respect to design variables are used to determine whether or not a specific natural frequency constraint is achievable. If there is a solution for a given frequency constraint, a solution existence result can be obtained very quickly using the method. Otherwise, the extreme value of the corresponding natural frequency or a small confined range of design variables which contains the extreme value can be obtained. Numerical examples are presented to illustrate the feasibility and efficiency of the proposed method.

Keywords: Solution existence; Truss structures; Frequency constraints

1. Introduction

The topic of structural optimization, subjected to static constraints such as stresses (Shanley, 1952; Schmit, 1960; Razani, 1965), displacements (Berke, 1970; Venkayya et al., 1973) and local stability (Khot et al., 1973, 1976), dynamic constraints such as natural frequencies (Turner, 1967; Zarghamee, 1968; Khot, 1985), frequency responses or other dynamic responses (Icerman, 1969; Johnson, 1974; Sadek, 1996), has been widely explored by many researchers. The common objective of the research is to

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construct algorithms to accomplish an optimal design, under the assumption that the solution of a given structural optimization problem exists. This assumption holds true for structural optimization with static constraints, unbounded continuously changing design variables, fixed topology configuration and material properties.

A structural optimization problem considering dynamic properties such as natural frequencies and dynamic responses may be termed a structural dynamic optimization problem (SDOP). SDOPs of some spring-mass systems and simple distributed mass systems have closed solutions (Wang, 1991). For the distributed mass systems, however, the solution existence for a given SDOP is often questionable. In fact, we come across now and then some SDOPs, which do not have a solution. For a practical SDOP, the optimal process is usually very computationally expensive. As the solution may not even exist, a meaningful solution may not be found after the intensive computation. Therefore, it is desirable to determine in advance whether there is a solution for a given SDOP.

The solution existence for a given SDOP is generally very difficult to determine. It depends on many parameters, such as topology, configuration, material, design variable linking, lower and upper bounds of design variables, discrete or continuous design variables, static and/or dynamic constraints, external loads, etc. In this work, we only consider SDOPs for truss structures with fixed topology, configuration and material properties. The sectional areas of the bars are chosen as design variables which can change continuously, and the design variable linking is defined at the beginning. The natural frequencies of the structure are chosen as constraints. A simple two-bar truss is first investigated, and a basic theory is then presented. A method based on this theory is proposed to examine more complex truss structures.

2. A simple example

Consider a two-bar planar truss, shown in Fig. 1. The material elastic modulus and density are 2.1×10^{11} Pa and 7.8×10^3 kg/m^3 for each bar. An optimization problem, demanding that the first eigenvalue be greater than 6.64×10^4 s^{-2}, the second eigenvalue less than 5.35×10^5 s^{-2} or greater than 8.42×10^5 s^{-2} is first examined. It can be proven that this problem does not hold a solution.

The eigenvalue-equation of the structure based on the finite element method can be written as:

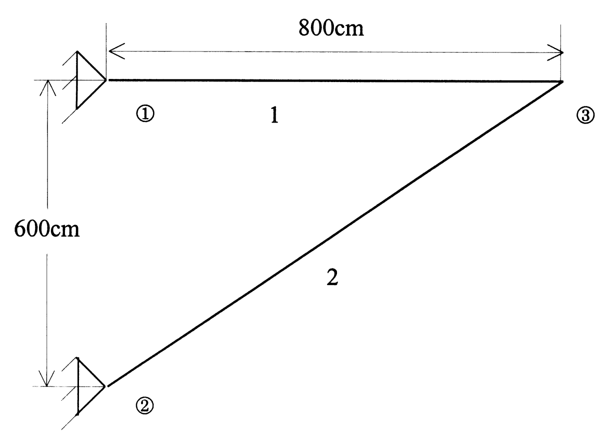

Fig. 1. A two-bar planar truss.

$$7.0 \times 10^3 \times \begin{bmatrix} 375A_1 + 192A_2 & 144A_2 \\ 144A_2 & 108A_2 \end{bmatrix} \{\phi_i\} = \lambda_i \times \begin{bmatrix} 3.12A_1 + 3.9A_2 & 0 \\ 0 & 3.12A_1 + 3.9A_2 \end{bmatrix} \{\phi_i\} \quad (1)$$

where A_1 and A_2 are the sectional-areas of bar 1 and bar 2, respectively, λ_i is the ith eigenvalue and $\{\phi_i\}$ is the corresponding eigenvector. Let $\lambda_i' = \lambda_i/10^4$ then the above equation can be rewritten as

$$(9.734\lambda_i'^2 - 8.190 \times 10^2\lambda_i')A_1^2 + (24.336\lambda_i'^2 - 1.679 \times 10^3\lambda_i' + 1.985 \times 10^4)A_2A_1 + (15.210\lambda_i'^2 - 8.190 \times 10^2\lambda_i')A_2^2 = 0.$$

For given A_2, Eq. (2) becomes a quadratic equation of A_1. For A_1 to have a real solution, it is required that

$$(24.336\lambda_i'^2 - 1.679 \times 10^3\lambda_i' + 1.985 \times 10^4)^2 - 4 \times (9.734\lambda_i'^2 - 8.190 \times 10^2\lambda_i') \times (15.210\lambda_i'^2 - 8.190 \times 10^2\lambda_i') \geq 0. \quad (3)$$

Thus we obtain

$$\lambda_1 \leq 6.639 \times 10^4, \quad \lambda_2 \geq 5.358 \times 10^5. \quad (4)$$

Therefore, it is impossible to have the first eigenvalue greater than 6.64×10^4 s^{-2} and the second eigenvalue less than 5.35×10^5 s^{-2}.

To examine other extreme values for the first and second eigenvalues, Eq. (2) is rewritten as

$$a\lambda_i'^2 + b\lambda_i' + c = 0 \quad (5)$$

where

$$a = 9.734A_1^2 + 24.336A_2A_1 + 15.21A_2^2$$

$$b = -(8.190 \times 10^2A_1^2 + 1.679 \times 10^3A_2A_1 + 8.190 \times 10^2A_2^2)$$

$$c = 1.985 \times 10^4A_2A_1 \quad (6)$$

We obtain the relationships between the two eigenvalues and the sectional areas of the bars:

$$\lambda_1 = \frac{b - \sqrt{b^2 - 4ac}}{2a} \times 10^4, \quad \lambda_2 = \frac{b + \sqrt{b^2 - 4ac}}{2a} \times 10^4. \quad (7)$$

It is found from Fig. 2 that the first eigenvalue becomes infinitely small when the section area of bar 2 is much larger than that of bar 1. Thus, the first eigenvalue of the two-bar truss does not hold a non-zero positive value as a minimum. When the sectional area of bar 2 is much less than that of bar 1, the second eigenvalue approaches a maximum, shown in Fig. 3. The maximum of the second eigenvalue can be found to be about 8.41×10^5 s^{-2} through further analysis. Thus it is impossible to have the second eigenvalue greater than 8.42×10^5 s^{-2}.

3. Basic theory

Definition. Assume that the cross-sectional areas of the bars of a truss before and after modification are

A_i and A_i' (i=1, 2, ..., m), respectively. If $(A_i' - A_i)/A_i = \alpha =$const, the modification on the truss is referred to as uniform modification.

3.1. Invariance of natural frequency

3.1.1. Statement: A uniform modification on a truss does not change its natural frequencies

This statement can be easily proven as follows.

The eigenvalue equation of an N degrees of freedom truss before modification can be written as

$$\det \left| [K] - \lambda_i [M] \right| = 0 \tag{8}$$

in which $[K]$ and $[M]$ are the stiffness and mass matrices of the truss structure, respectively, and λ_i is the ith eigenvalue of the structure.

After a uniform modification, the eigenvalue problem equation can be rewritten as

$$\det \left| [K'] - \lambda_i [M'] \right| = 0 \tag{9}$$

where

$$[K'] = \sum_{i=1}^{m} (1+\alpha) A_i [K^i] = (1+\alpha)[K] \tag{10}$$

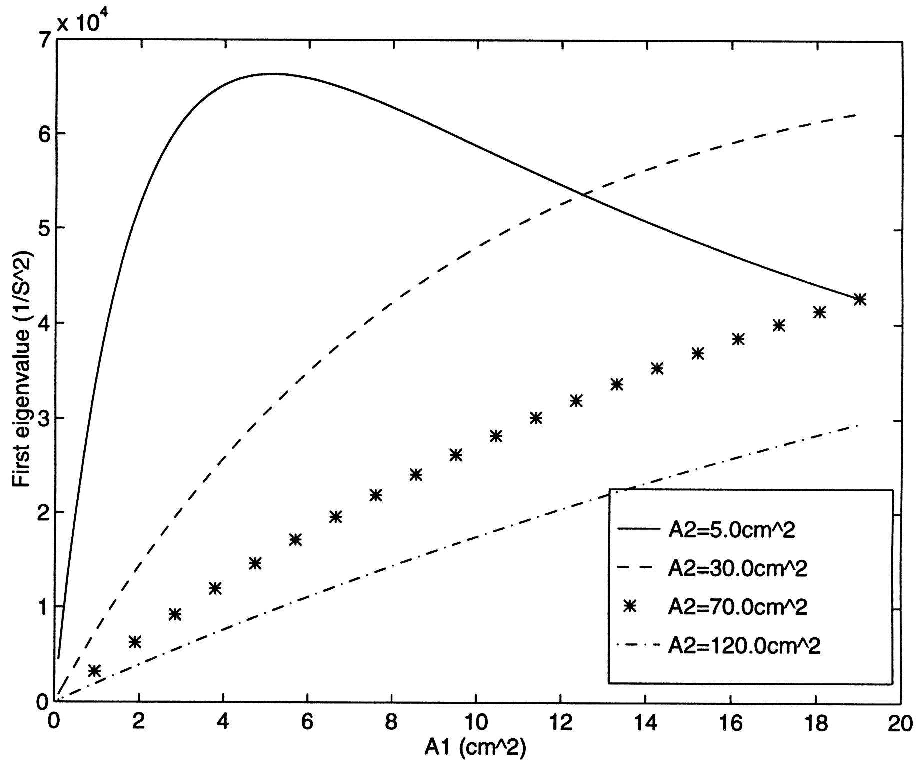

Fig. 2. First eigenvalue of the two-bar truss.

$$[M'] = \sum_{i=1}^{m} (1+\alpha) A_i [M^i] = (1+\alpha)[M] \tag{11}$$

in which $[K^i]$, $[M^i]$ are matrices related only to the ith bar with a unit area. Substituting Eqs. (10) and (11) into Eq. (9), it can be found that Eq. (9) is identical to Eq. (8), which means that the eigenvalue-problem of the truss after a uniform modification is unchanged, and therefore the statement is proven.

The statement is used in the following sections to examine the key constraint that determines whether there is a solution for a truss SDOP.

3.2. Key constraint for truss optimization problem

3.2.1. Statement: the natural frequency constraint is the key constraint to determine the solution existence of a truss dynamic optimization problem

In 1976, Johnson studied the two-dimensional design space for the steady-state response of a periodically loaded structure and revealed that the feasible design space was disconnected. He also pointed out that the cause of the disjoint feasible domain was the resonance of natural frequencies with the loading frequencies. The disjoint property implies that the external loading can affect the properties of the feasible domain of the optimization problem due to resonance. To avoid the resonance effect and simplify the discussion, we assume: if structural damping is ignored, the frequencies of loading are far from the natural frequencies; if structural damping is considered, it is Rayleigh damping.

In a truss SDOP, constraints may be stresses, displacements, frequency responses, impulse responses,

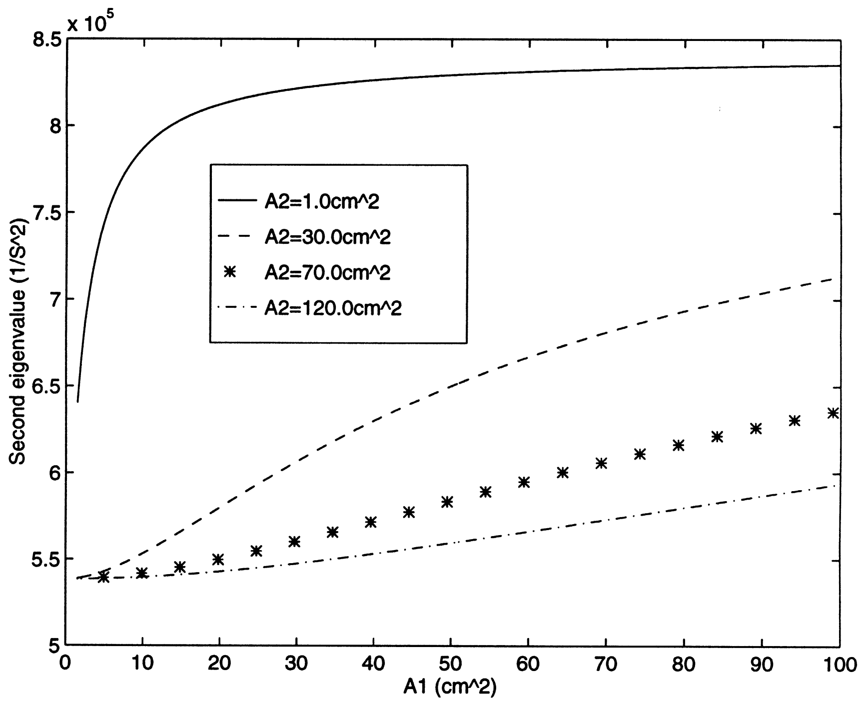

Fig. 3. Second eigenvalue of the two-bar truss.

random excitation responses, lower and upper bounds of design variables, natural frequencies or their combinations. The key constraint, which determines the existence of a solution, is generally frequency constraint. This can be justified through a minimum weight truss SDOP, which has constraints of stresses, lower bounds of design variables and natural frequencies. Truss SDOPs with other constraint combinations can be verified in a similar manner.

The existence of solution of a SDOP depends on the properties of its constraints' feasible domain. When the constraints do not have a feasible domain, obviously, the optimization problem does not have a solution; when the constraints' feasible domain is a continuous and closed region, the solution of the SDOP exists. It is obvious that the lower bound constraints of design variables have a feasible domain, and the domain is convex.

If the dynamic stress constraint in each bar can be satisfied at a certain design point, then such point must be a point within the stress constraints' feasible domain, thus, its feasible domain exists. Suppose that some bars' dynamic stress constraints are not satisfied at the current design point, or the point is not within the feasible domain of stress constraints. Because the design variables can change continuously and their upper bounds are not considered, there must exist a feasible design point, such as the point obtained by a uniform modification which satisfies each stress constraint.

When a truss is modified uniformly at a certain ratio and the external forces are unchanged, the axial force in any bar of the truss stays unchanged and stress in a bar of the truss varies inversely at the same ratio. Therefore, if design variables can be modified continuously and their upper bounds are not considered, stress constraints must exist in a feasible domain, such as the domain $\{A_i$: $A_i \geq A_i'$, where $i = 1, 2, \ldots, m$, A_i' is the area at which all of the stress constraints are satisfied, m is the number of design variables$\}$. This domain is a stress constraints' feasible domain for a statically determinate truss. For a statically indeterminate truss with design variable linking, there must exist a set of design variable values for given external loading. When the design variables are greater than these values, all of the stress constraints can be satisfied, thus, stress constraints have a feasible domain, although any variation of a design variable may result in a reassignment of internal forces.

The feasible domain can become closed when a suitable upper bound is given to each design variable. Thus, under the constraints of stress and design variable lower bounds, a feasible solution for the truss minimum weight optimization problem must exist.

On the other hand, Eq. (8) can be rewritten as:

$$\det\left|\sum_{i=1}^{m} u_i[K^i] - \lambda_j \sum_{i=1}^{m} u_i[M^i]\right| = 0 \quad j = 1, 2, \ldots, n \tag{12}$$

where $[K^i]$ and $[M^i]$ are the stiffness and mass matrices with unit values for the design variables, respectively, u_i is the corresponding design variable, λ_j is the selected eigenvalue to be constrained, m is the number of design variables, n is the number of natural frequencies to be constrained.

Eq. (12) is a set of n non-linear equations with m variables. If Eq. (12) has a solution, it must have infinite number of solutions because a uniform modification on a truss does not change its natural frequencies. In those solutions there must exist a solution, which can satisfy the design variable lower bound and stress constraints simultaneously. Thus, when natural frequency constraints have a feasible domain, the feasible domain of design variable lower bound and stress constraints must also exist. Therefore, we conclude that the key constraint for a truss SDOP is the frequency constraint.

Based on the theory proven above, a method which only examines the natural frequency constraint will be presented in the following section to determine the solution existence of truss SDOPs.

4. Method and algorithm

Generally speaking, there are three types of natural frequency constraints:

$$\lambda_1 \geq \lambda_1^{l} \tag{13}$$

$$\lambda_i \leq \lambda_i^{u} \quad \text{and} \quad \lambda_{i+1} \geq \overline{\lambda_{i+1}} \tag{14}$$

$$\lambda_j = \overline{\lambda_j}. \tag{15}$$

Eq. (13) indicates that the fundamental frequency should not be less than a certain value, Eq. (14) indicates restricted zone constraints for some natural frequencies and Eq. (15) indicates that several natural frequencies are demanded to satisfy specific values.

The solution existence of a SDOP depends on whether or not the specific values, e.g. λ_1^{l}, $\overline{\lambda_{i+1}}$, are beyond their extreme values. If the specific value for a given constraint is within its extreme value, the constraint is achievable in practical optimization, and if all the natural frequency constraints are achievable, the SDOP holds a solution. Therefore, to determine the solution existence of a SDOP, basically, what has to be done is to quickly optimize the corresponding eigenvalue. The objective function of the optimization is the corresponding eigenvalue and the constraints are on design variables. For constraints of $\lambda_1 \geq \lambda_1^{l}$, $\lambda_{i+1} \geq \overline{\lambda_{i+1}}$ and $\lambda_j = \overline{\lambda_j}$ (if $\lambda_j^0 < \overline{\lambda_j}$, λ_j^0 is the jth eigenvalue of the initially designed structure), a maximization, and for constraints of $\lambda_i \leq \lambda_i^{u}$ and $\lambda_j = \overline{\lambda_j}$ (if $\lambda_j^0 > \overline{\lambda_j}$), a minimization for the corresponding eigenvalue are to be carried out.

The maximization can be done through the following procedure.

Maximize

$$\lambda_i = \lambda_i(u_k) \quad k = 1, 2, \ldots, m \tag{16}$$

Subjected to

$$u_k \geq 0 \quad k = 1, 2, \ldots, m. \tag{17}$$

Using the steepest decent method (SDM), we start from the initial point u_k^0 and iteratively move towards the maximum point according to the rule

$$u_k = u_k^0 + \alpha v_k \quad k = 1, 2, \ldots, m \tag{18}$$

$$v_k = \partial \lambda_i / \partial u_k \tag{19}$$

where α is the iterative step size in the direction $V = (v_1, v_2, \ldots, v_m)$. And v_k can be obtained by (Fox and Kapoor, 1968)

$$v_k = \frac{\{\phi_i\}^{\mathrm{T}} \left(\dfrac{\partial [K]}{\partial u_k} - \lambda_i \dfrac{\partial [M]}{\partial u_k} \right) \{\phi_i\}}{\{\phi_i\}^{\mathrm{T}} [M] \{\phi_i\}} \tag{20}$$

in which $\{\phi_{\mathrm{i}}\}$ is the ith eigenvector.

For the conventional SDM, the step size α is determined according to the optimal solution to the line-search problem of minimizing $\lambda_i(u_k + \alpha v_k)$ subjected to $\alpha \geq 0$. As the objective is the eigenvalue, a modal analysis must be used and the analysis is computationally expensive, Eq. (18) is therefore modified as

$$u_k = u_k^0 + \beta\overline{v_k} \quad k = 1, 2, \ldots, m \tag{21}$$

in which

$$\overline{v_k} = v_k / \sqrt{v_1^2 + v_2^2 + \cdots + v_m^2} \tag{22}$$

$$\beta = \frac{c_0}{\beta_0} \tag{23}$$

$$c_0 = \max\left(\frac{u_p}{\overline{v_p}}\right) \quad \text{for all} \quad v_p < 0 \quad p = 1, 2 \ldots, m \tag{24}$$

where $\beta > 1$ is a positive weight coefficient determined in the iterative process, as shown in Fig. 4. Eq. (22) is employed to normalize the moving direction, and Eqs. (23) and (24) are used to determine the step size and to ensure the feasibility of modification on the structure, respectively.

Using the above method, the sub-process of optimization for determining the step size β can be omitted. If the specific value of the natural frequency constraint, λ_1^1, $\overline{\lambda_{i+1}}$, is far from its extreme value and can be satisfied, the convergence will be fast because the step size is large and the moving direction is steepest. If the specific value is beyond its extreme value, the corresponding natural frequency can also quickly approach the neighborhood of its extreme value. When the natural frequency has approached the neighborhood of its extreme value, the step size determined by the above method may be too large and thus the corresponding frequency may move toward the opposite direction. Fortunately, it is very easy to determine whether there is an extreme value along a design variable. This is because when there is an extreme value, either its sensitivity is very small or the sign of it changes within a small range. Therefore, if the specific value of the natural frequency constraint is beyond or within the neighborhood of its extreme value, we can determine the extreme value or a very small range of design variables which contains the extreme value using the following equation:

$$u_k = \frac{1}{2}(u_k^i + u_k^{i+1}) \quad \text{for} \quad \text{sign}(v_k^i) = -\text{sign}(v_k^{i+1}). \tag{25}$$

Where u_k^i, u_k^{i+1} and v_k^i, v_k^{i+1} are the values of u_k and v_k at the ith and $(i + 1)$th iterations, respectively. To terminate the iterative process, all of the following optimality criteria are used:

$$\lambda_i \geq \overline{\lambda_i} \tag{26}$$

$$\left|\frac{\partial \lambda_i}{\partial u_k}\right| \leq \varepsilon_1 \quad k = 1, 2, \ldots, m \tag{27}$$

$$|\, u_k^i - u_k^{i+1} \,| \leq \varepsilon_2 \quad \text{and} \quad \text{sign}(\overline{v_k^i}) = -\text{sign}(\overline{v_k^{i+1}}) \quad k = 1, 2, \ldots, m \tag{28}$$

When inequality (26) is satisfied, the corresponding natural frequency constraints can be satisfied, and when inequality (27) is satisfied, the corresponding natural frequency approaches its maximum. When Eq. (28) is satisfied, a small domain of design variables which contain the maximum value of the corresponding eigenvalue is obtained. In Eqs. (27) and (28), ε_1 and ε_2 are the error control parameters, pre-determined by the accuracy desired. The flow chart shown in Fig. 4 illustrates the proposed method.

For SDOPs of large structures, the sub-structure synthesis method and approximate reanalysis techniques can be used to speed up the iteration process.

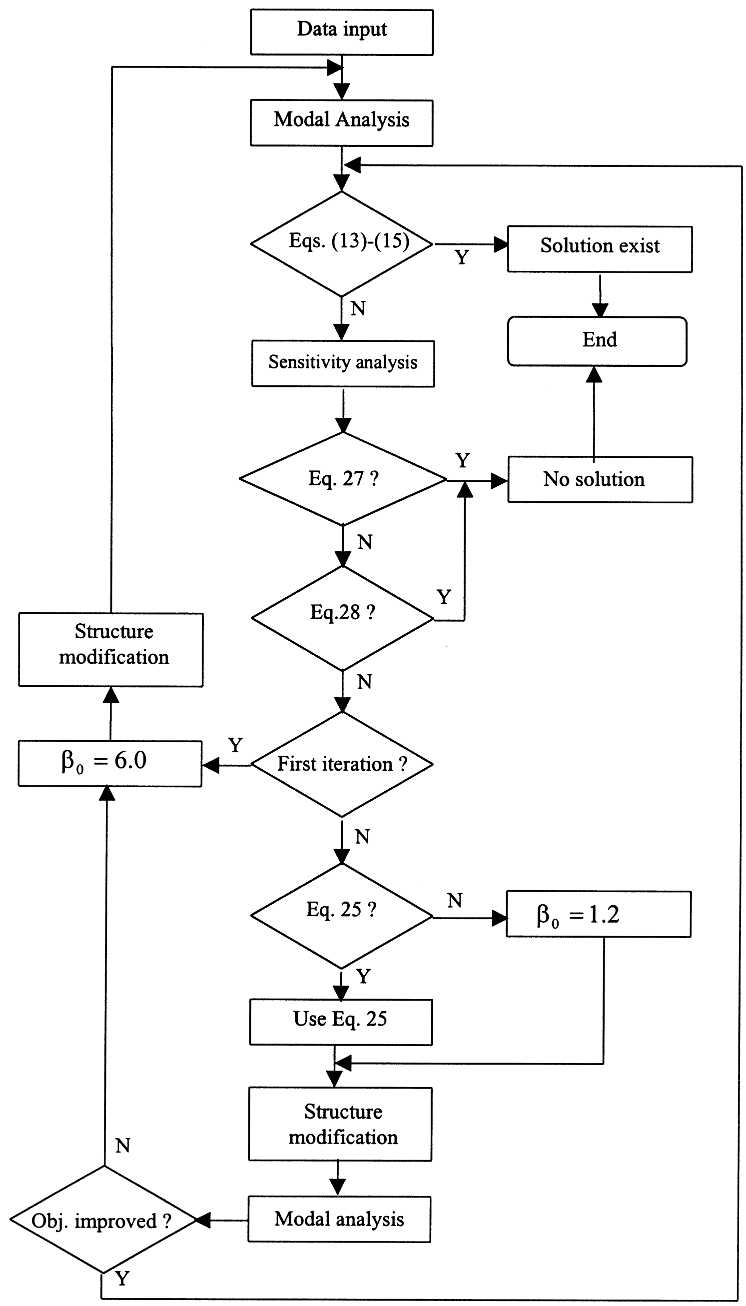

Fig. 4. Flow chart for determining the solution existence.

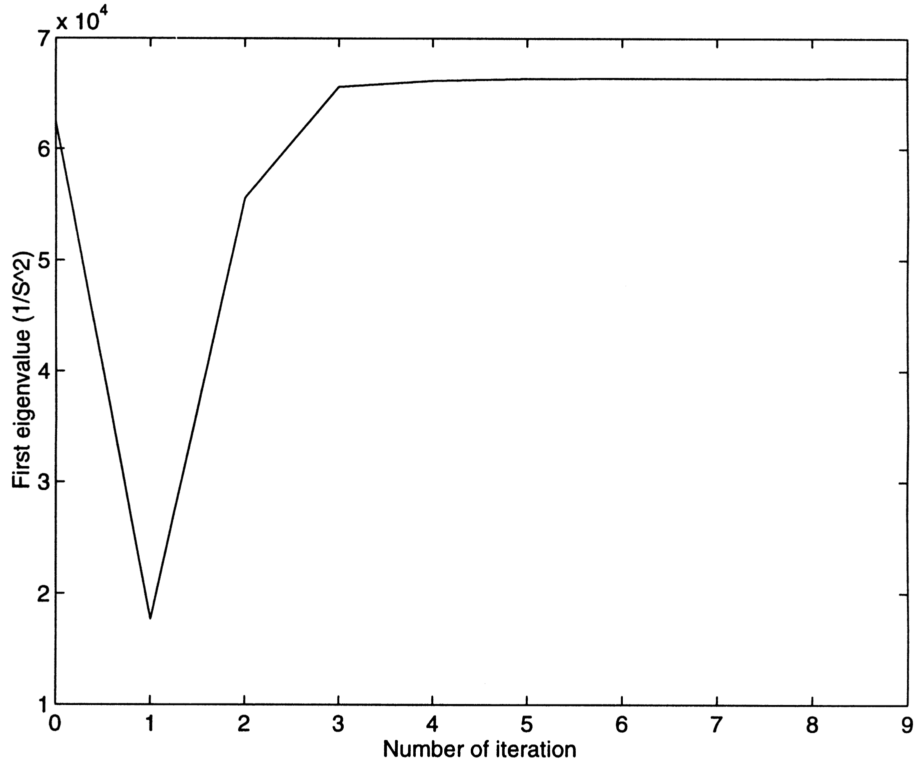

Fig. 5. Iteration history of the first eigenvalue of the two-bar truss ($\lambda_1 \geq 7.0 \times 10^4$).

5. Numerical example

Examples for three truss structures are discussed in this section. The material properties for these trusses are the same as those given in section 2.

5.1. Two-bar truss problem

Fig. 1 shows the configuration of a two-bar planar truss. The initial sectional areas of bar 1 and 2 are 6.222 cm^2 and 10.208 cm^2, respectively. If the natural frequency constraint for the optimization is that the first eigenvalue is not less than 6.0×10^4 s^{-2}, no iteration is needed to determine the existence of solution because the first eigenvalue at the initial point is already found to be greater than 6.0×10^4 s^{-2}. Suppose we constrain the first eigenvalue to be not less than 7.0×10^4 s^{-2}, only nine iterations are needed to reach the non-solution conclusion and to obtain a small range of design variables which contain the maximum value of the first eigenvalue, as shown in Fig. 5 and Table 1. It can also be seen from Fig. 5 that the first eigenvalue is between 6.0×10^4 s^{-2} and 7.0×10^4 s^{-2}. It tallies well with the theoretical value of 6.64×10^4 s^{-2}.

If the natural frequency constraint is given for the second eigenvalue not to be greater than 5.3×10^5 s^{-2}, non-existence of solution output can be obtained at the 3rd iteration, and the minimum value of the second eigenvalue is found to be greater than 5.35×10^5 s^{-2} which is the same as the theoretical value given in section 2. The iterative history is shown in Fig. 6.

Table 1
Design variable range containing the maximum value of the first natural frequency for the two-bar truss

	A_1 (cm^2)	A_2 (cm^2)
Lower bound	8.5117	8.8123
Upper bound	8.5662	8.7792

Table 2
Design variable linking of the ten-bar planar truss

Design variable no.	1	2	3	4
Bar no.	1,2	3,4	5,6	7,8,9,10

5.2. Ten-bar truss problem

Fig. 7 shows the configuration of a ten-bar planar truss. The initial sectional area of each bar is 10 cm^2. Ten bars are divided into four design variable linking sets shown in Table 2. Suppose the natural frequency constraint for the optimization is that the first natural frequency is not less than 50.0 Hz, only three iterations determine the existence of solution, as shown in Fig. 8.

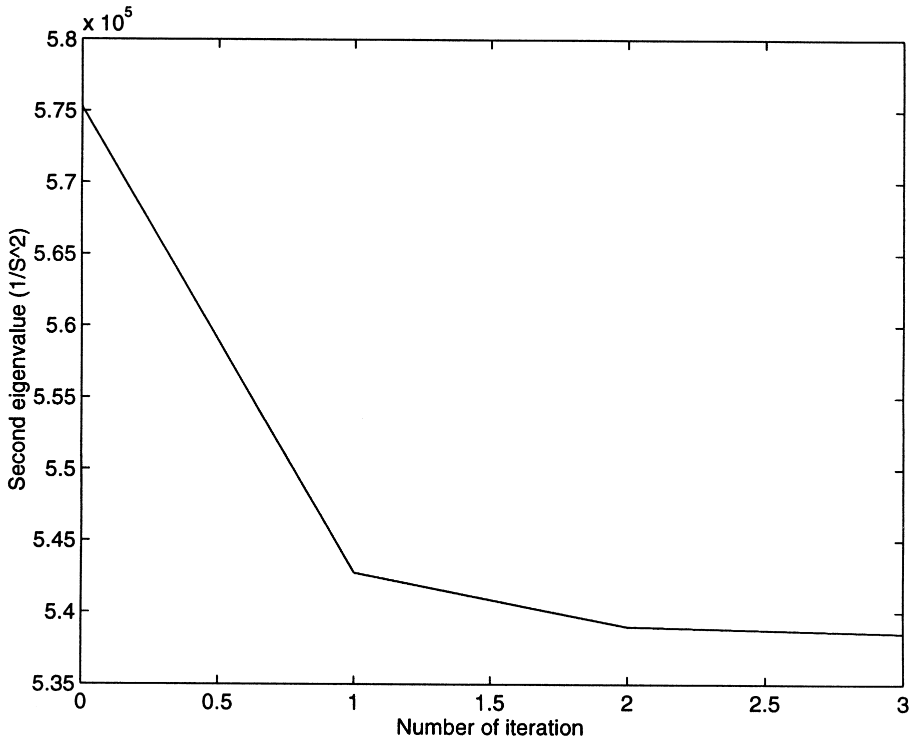

Fig. 6. Iteration history of the second eigenvalue of the two-bar truss ($\lambda_2 \leq 5.3 \times 10^5$).

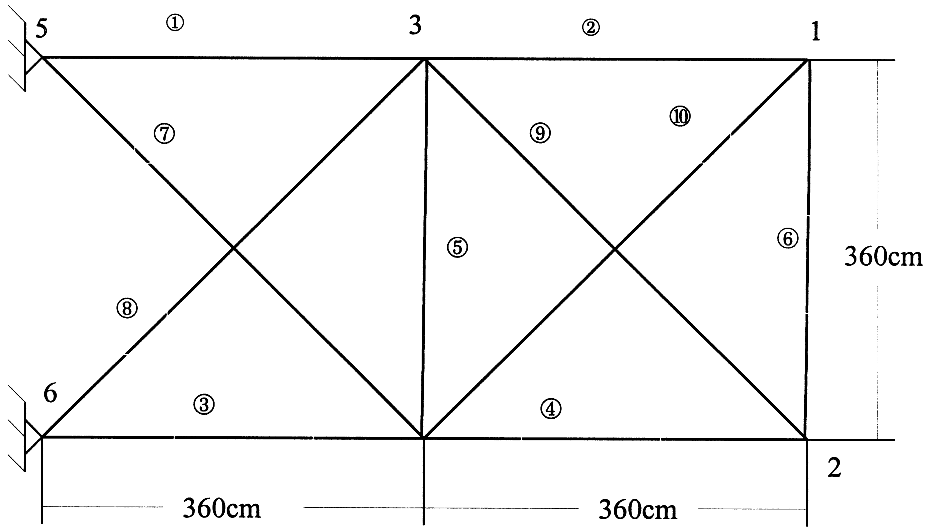

Fig. 7. A ten-bar planar truss.

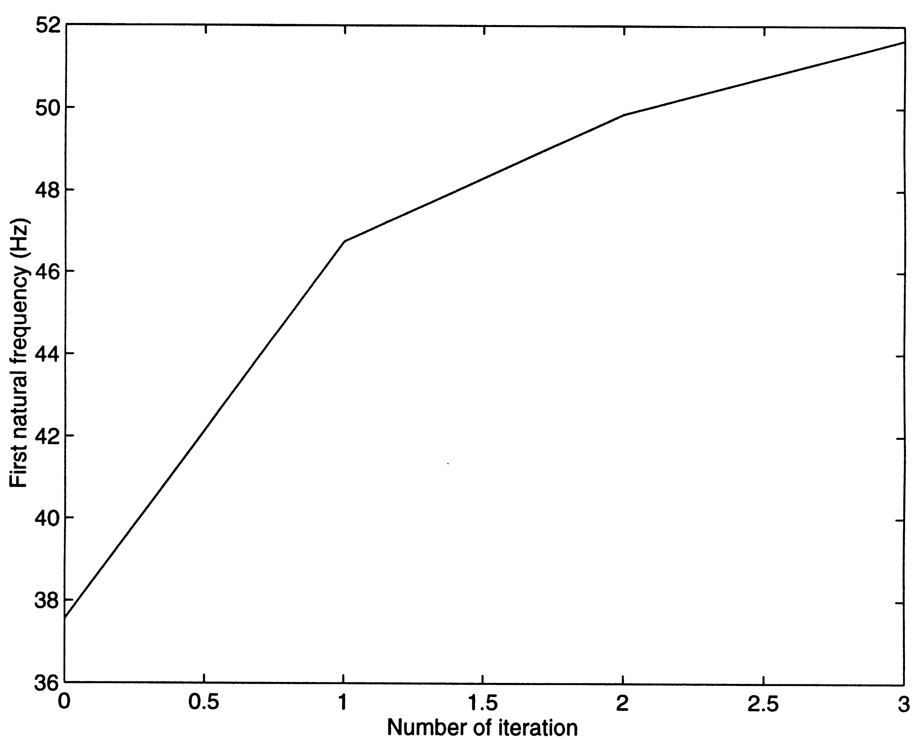

Fig. 8. Iteration history of the first frequency of the ten-bar truss ($f_1 \geq 50.0$ Hz).

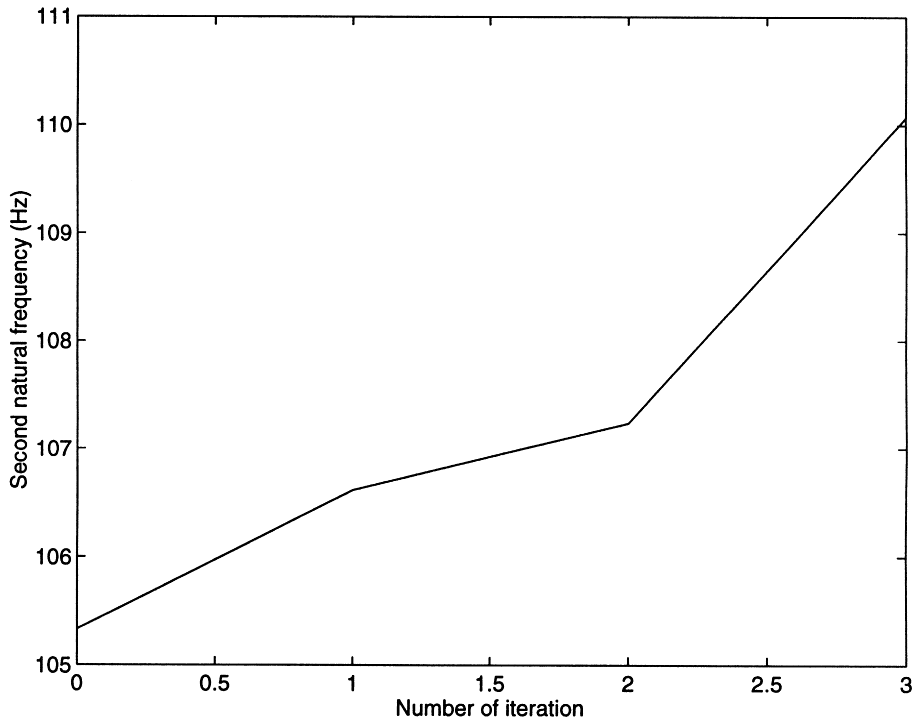

Fig. 9. Iteration history of the second frequency of the ten-bar truss ($f_2 \geq 109.9$ Hz).

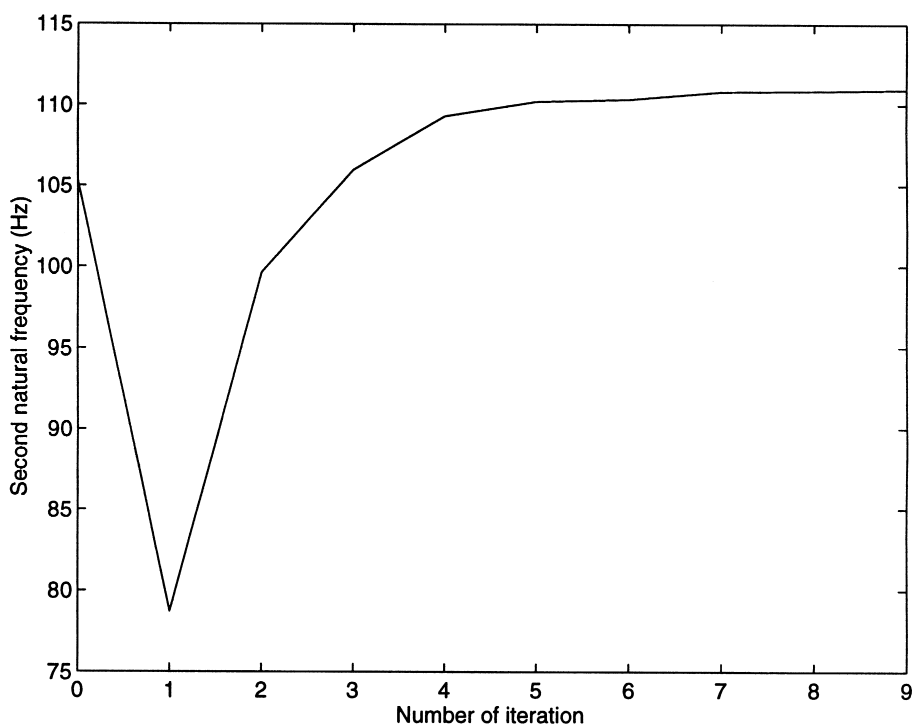

Fig. 10. Iteration history of the second frequency of the ten-bar truss ($f_2 \geq 115.0$ Hz).

Table 3
Design variable range containing the maximum value of the second natural frequency for the ten-bar truss

	u_1 (cm^2)	u_2 (cm^2)	u_3 (cm^2)	u_4 (cm^2)
Lower bound	9.9013	8.2312	5.8333	10.736
Upper bound	9.9065	8.2521	5.8659	10.777

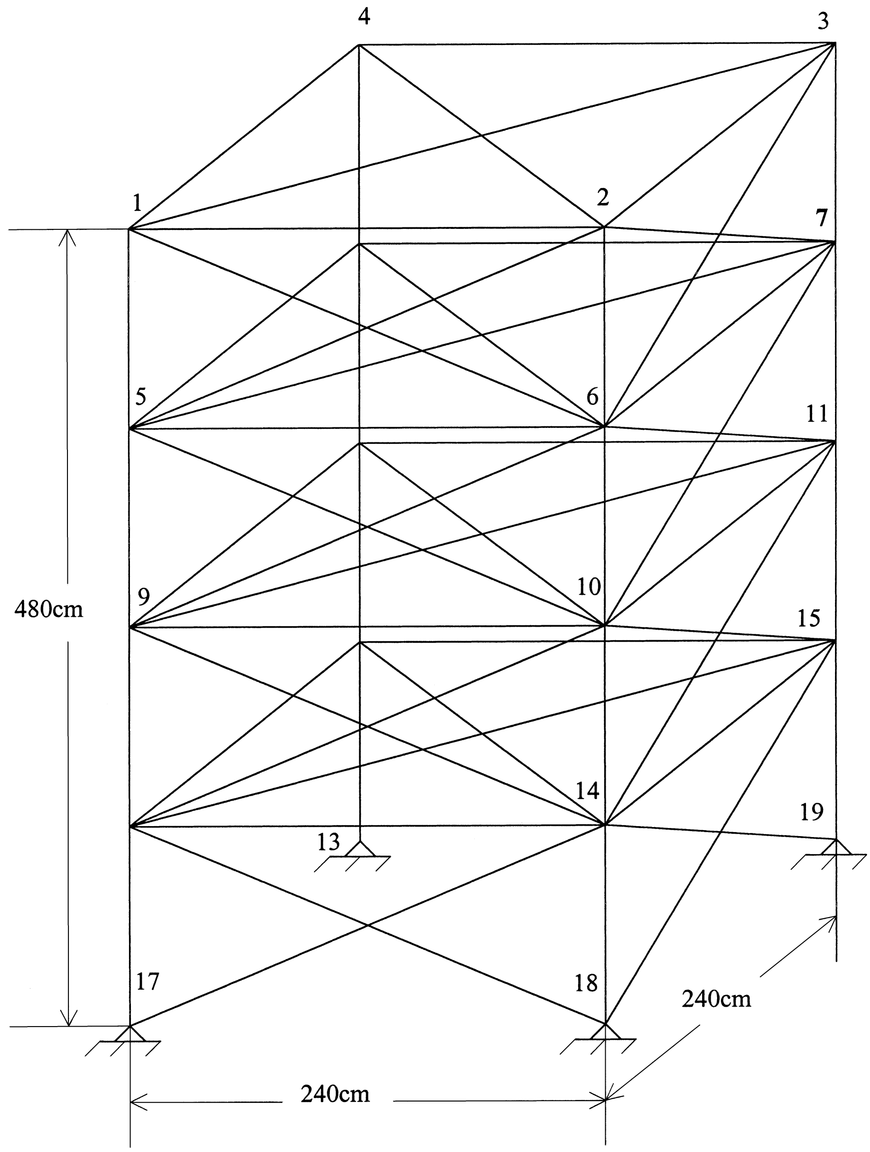

Fig. 11. 72-bar space truss.

Table 4
Design variable linking of the 72-bar space truss

Design variable number	Bar number
1	1–4
2	5–12
3	13–16
4	17,18
5	19–22
6	23–30
7	31–34
8	35,36
9	37–40
10	41–48
11	49–52
12	53,54
13	55–58
14	59–66
15	67–70
16	71,72

If the natural frequency constraint is given for the second natural frequency not to be less than 109.9 Hz, the iteration process history of the second natural frequency is plotted in Fig. 9. Only three

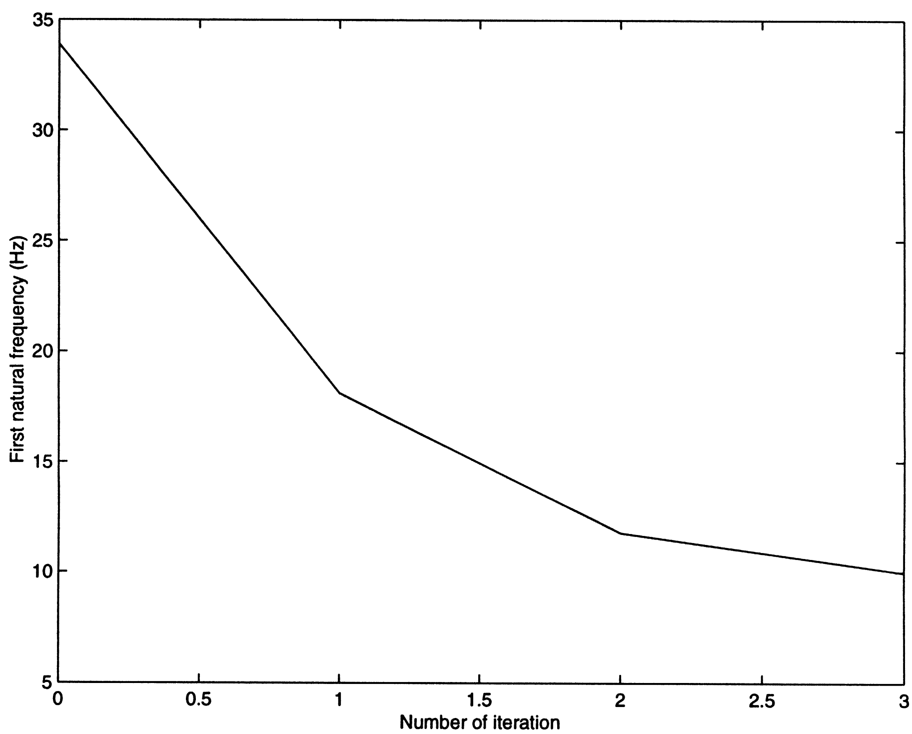

Fig. 12. Iteration history of the first frequency of the 72-bar truss ($f_1 \leq 10.0$ Hz).

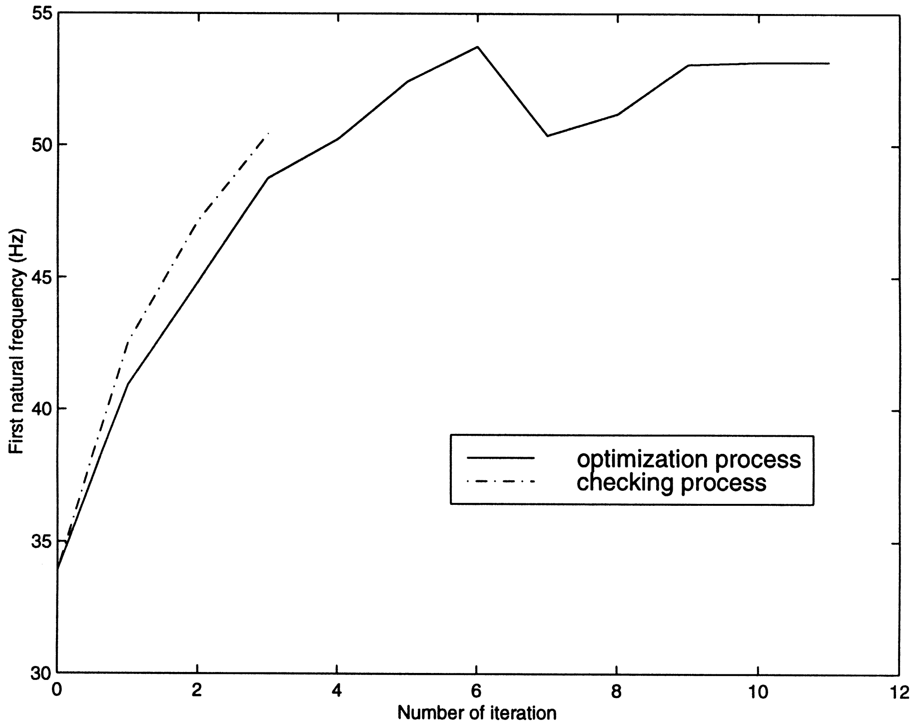

Fig. 13. Iteration history of the first frequency of the 72-bar truss ($f_1 \geq 50.0$ Hz).

iterations are needed to determine the existence of solution. If the target value for the second natural frequency is set at 115.0 Hz, only nine iterations determine the non-existence of solution, and a small range of design variables which contain the maximum value of the second natural frequency, as shown in Fig. 10 and Table 3.

5.3. 72-bar truss problem

Fig. 11 shows the configuration of a 72-bar space truss. The 72 bars are divided into 16 design variables as shown in Table 4. The initial design of the structure has cross sectional area of 10 cm^2 for each bar. If the natural frequency constraint for the optimization is that the first natural frequency is not greater than 10.0 Hz, three iterations determine the existence of solution, as shown in Fig. 12. If the target value for the first natural frequency is set to be no less than 50.0 Hz, three iterations can also determine the existence of solution, as shown in Fig. 13.

To show the efficiency of the proposed method, a practical SDOP of 72-bar truss is considered. The

Table 5
Initial and optimum characteristics of the 72-bar truss

	f_1 (Hz)	A_{44} (M/N)	A_{84} (M/N)	Weight
Initial	33.92	7.05×10^{-7}	8.45×10^{-9}	1609.4 kg
Optimum	53.17	1.94×10^{-7}	3.50×10^{-9}	311.5 kg

objective of the optimization is to minimize the weight of the structure. The initial parameters of the structure are the same as section 5.3. Two load conditions are assumed. In the first load case, three static external loads of 2.223×10^4 N are imposed on node 1 in the x, y and -z directions. In the second load case, one harmonic external force with frequency 18.0 Hz is assumed to load at the 4th DOF. The allowable tensile and compressive stresses are assumed to be 172.4 MPa. The lower bound of the design variable is 6.452 cm^2. The natural frequency constraint demands that the fundamental frequency of the structure is no less than 50.0 Hz. The response constraints demand that the frequency response amplitudes at 4th and 8th DOF are no greater than 2.0×10^{-7} M/N. Using Silicon Graphics workstation, it takes 12.3 s CPU time to obtain the optimum solution, the initial and optimum characteristics of the structure are shown in Table 5. However, to determine the solution existence of the optimization problem, only 0.3 s CPU time was needed. The CPU time used for checking process is only about 2.4% of that for the actual optimization.

6. Conclusion and discussion

A basic theory has been presented and proven to determine the key constraint of frequency optimization problems of truss structures. Based on this theory a practical method has been proposed. The proposed method is efficient, practically useful for determining the solution existence of truss SDOPs. If a solution exists for a truss SDOP, the checking process is very fast and the data of modal analysis and sensitivity analysis can be utilized in the following actual optimization. The CPU time used for determining the solution existence is only a very small portion of the time needed for the actual optimization. If there is no solution for a given SDOP, the cost for checking the solution existence is easily justifiable, as this can save the loss in a blind actual optimization, which can be very costly in most practical applications.

In an optimization process, a different design point will sometimes cause the optimization process to converge at different local optima. However, the starting point for the solution existence searching should not affect the outcome of the solution existence. The reason is that in the present searching procedure, the checking is performed for each individual eigenvalue separately. The specified ith eigenvalue can have at most one minimum and one maximum value when the design variables vary continuously (see, Figs. 2 and 3, as examples).

The existence of a solution for a SDOP depends on the properties of its constraints' feasible domain. This paper revealed that the natural frequency constraint is the key constraint to determine the solution existence of a truss SDOP. For more complicated structure rather than trusses, the solution existence of a SDOP can be much more complicated. The natural frequency constraints may not be the key constraint to determine the solution existence of the structure. The present primary research provides a direction for further research on complex structures.

In the checking process, a modal analysis with each iteration is needed to determine the convergence. Utilizing approximation methods is preferable in practical checking process to avoid performing a full eigenvalue analysis with each iteration. The present method requires a separate checking for each frequency constraint which may be costly for problems with large numbers of frequency constraints, however, the time is still much smaller than an actual full optimization.

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